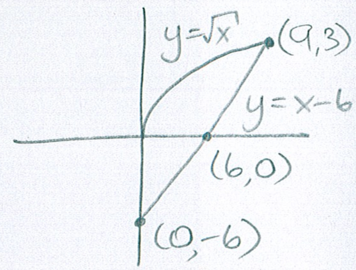
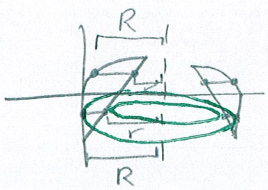
Consider the region bounded by $y=\sqrt{x}$, $y=x-6$ and $x=0$. **SCORE: _____ / 14 PTS**



[a] If the region is revolved around the line $x=12$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

[i] using a dy integral **(NOTE: You do NOT need to simplify your integrand.)**



$$\pi\int_{-6}^{0}\left((12-0)^2-(12-(y+6))^2\right)dy$$

$$+$$

$$\pi\int_{0}^{3}\left((12-y^2)^2-(12-(y+6))^2\right)dy$$

$y=\sqrt{x} \rightarrow x=y^2$

$y=x-6 \rightarrow x=y+6$

$\sqrt{x}=x-6$

$x=x^2-12x+36$

$0=x^2-13x+36$

$0=(x-4)(x-9)$

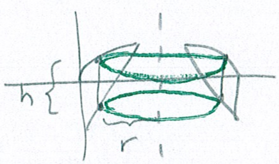
$x=4,9$

$\downarrow$

$y=x-6=\cancel{-2},3$

1 2 2 1 1 1

[ii] using a dx integral **(NOTE: You do NOT need to simplify your integrand.)**



$$2\pi\int_{0}^{9}(12-x)(\sqrt{x}-(x-6))\,dx$$

1 1 1 1

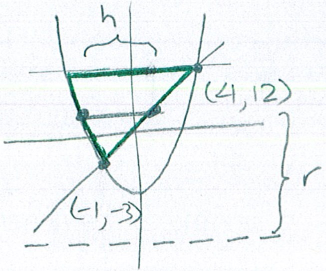
[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{\pi}{8}\int_{0}^{9}(\sqrt{x}-(x-6))^2\,dx$$

1 $\frac{1}{2}$ 1

The region defined by $y \geq x^2 - 4$, $y \geq 3x$ and $y \leq 12$ is revolved around the line $y = -8$. **SCORE: _____ / 8 PTS**

Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid **using as few integrals as possible**.



$$\underbrace{2\pi}_{1} \overbrace{\int_{-3}^{12}}^{2} \underbrace{(y - -8)}_{2} \underbrace{\left(\tfrac{1}{3}y - -\sqrt{y+4}\right)}_{3} dy$$

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1, 4$

$\downarrow \quad \downarrow$

$y = -3, 12$

TALK TO ME IF YOU WROTE
dx WASHER METHOD INTEGRALS

$y = x^2 - 4 \longrightarrow x = \pm\sqrt{y+4}$ $\quad x = -\sqrt{y+4}$ FOR LEFT SIDE OF PARABOLA

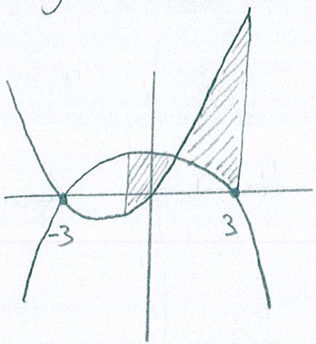
$y = 3x \longrightarrow x = \frac{1}{3}y$

Find the area between the curves $y = 36 - 4x^2$ and $y = 2x^2 + 6x$ over the interval $[-1, 3]$. **SCORE: _____ / 8 PTS**

NOTE: Your final answer must be a number, not an integral nor sum of integrals.

$y = 36 - 4x^2 \to x\text{-INT} = \pm 3$

$y = 2x^2 + 6x \to x\text{-INT} = 0, -3$



$36 - 4x^2 = 2x^2 + 6x$

$0 = 6x^2 + 6x - 36$

$0 = 6(x^2 + x - 6)$

$0 = 6(x+3)(x-2)$

$x = -3, 2$

$1\frac{1}{2}$ $\quad 1 \quad$ $1\frac{1}{2}$ $\quad 1$

$$\left[\int_{-1}^{2} \underline{(36 - 4x^2 - (2x^2 + 6x))}\,dx\right] + \left[\int_{2}^{3} \underline{(2x^2 + 6x - (36 - 4x^2))}\,dx\right]$$

$$= \int_{-1}^{2} (36 - 6x - 6x^2)\,dx + \int_{2}^{3} (6x^2 + 6x - 36)\,dx$$

$1 \quad 1$

$$= \underline{(36x - 3x^2 - 2x^3)}\Big|_{-1}^{2} + \underline{(2x^3 + 3x^2 - 36x)}\Big|_{2}^{3}$$

$$= 36(2 - {-1}) - 3(4 - 1) - 2(8 - {-1})$$

$$+ 2(27 - 8) + 3(9 - 4) - 36(3 - 2)$$

$$= 108 - 9 - \underset{-3}{\cancel{18}} + \underset{2}{\cancel{38}} + \cancel{15} - \cancel{36}$$

$$= \underline{98}$$

1